**Standard and Internal Modelling Under Solvency II**

*How Stochastic Modelling can contribute to better risk management*

## Introduction

The main goal of the upcoming Solvency II legislation for insurers is to strengthen guarantees to policyholders. An improved alignment of capital requirements with risk exposure should significantly contribute towards this goal.

The assessment of the required solvency margin and financial position of an insurer are among the main areas of expertise of the actuarial profession. In this article we will show how stochastic modelling as part of an internal model approach can help improve such assessment. In a comparison with the standard model for a sample Life Insurance company, advantages of the stochastic model approach are shown.

## 1 Modelling of Extreme Events

Solvency II legislation will require insurance companies to have capital buffers that ensure solvency over a one year time horizon with a probability of at least 99.5%. The occurrence of an event leading to insolvency of an insurer can therefore be called extreme.

Without taking a position whether the 99.5% criterion indeed provides sufficient assurance for policyholders, we observe the following:

Before the credit crisis, financial companies often underestimated the likelihood of extreme events. As a board member of Goldman Sachs stated: *August 2007 was a very special month. Things were happening then that were only supposed to happen about once in every 100,000 years. Either that… or Goldman's models were wrong.*

We need to realise that modelling assumptions that appear reasonable under normal circumstances may nevertheless prove inappropriate under more extreme conditions. Such assumptions may apply to individual risks, as well as the dependence between them.

In particular, market risks tend to show so-called tail-correlation, i.e. the correlation between asset price movements increases under stressed conditions. In 2008, the former chairman of the Federal Reserve Alan Greenspan commented that '*negative correlations among asset classes, so evident during an expansion, can collapse as all asset prices fall together, undermining the strategy of improving risk/reward trade-offs through diversification.*'[1]

[1] Financial Times 16 March 2008

## 2 Solvency II Standard Model

The Solvency II standard model as put forward in QIS 5 covers a wide and diverse range of risks, and reflects an industry-wide perspective on the magnitude of those risks. As such it is not an unreasonable starting point for a Solvency assessment by an external regulator.

However, for any individual company, there can be large deviations between the actual risk profile, and the risk profile assumed in the standard model. On the one hand, such deviations may be attributed to the choice of model parameters. In that case, the company specific parameters can be used as input to the standard model.

On the other hand, the structure of the standard model gives rise to deviations that can not be resolved by a change of parameters. An example is the aggregation methodology, the method used to determine the capital for all risks in aggregation, based on individual risks and their dependencies.

The aggregation methodology in the standard model makes use of various implicit assumptions. One of these is the additivity of risks, i.e the aggregated risk is the sum of the individual risks, taking into account the dependencies between them. This is an assumption that does not hold true in many cases. For example:

> *Suppose a Life Insurance portfolio has a much higher than expected Lapse rate in a certain year As a result, the portfolio size declines, and therefore, so does the Mortality risk in the remaining portfolio. Hence the combined risk of Mortality and Lapse is not well represented by the sum of the two separate risks. A higher Lapse rate leads to lower Mortality risk, even if there is no correlation between Mortality and Lapse.*
>
> *Much in the same way, there is a non-additive relation between Mortality and Interest rate risk. When interest rates fall, the present value of any future cashflow increases. As a result, the mortality risk in terms of the uncertainty of the present value of future benefit payments, increases with a decline in interest rates.*

Except for the additivity of risks, other implicit assumptions of the standard model that are often not satisfied are:

- In risk aggregation, all risks and combinations of risks need to follow a Normal distribution[2].
- Correlations between risks are assumed not to increase under extreme circumstances. Especially for market risk, this assumption tends to be unrealistic.

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[2] Strictly speaking, Elliptic distributions other than Normal distributions also satisfy the assumptions of the correlation matrix approach of the standard model. However, because of their special shape and dependence structure these distributions are of extremely limited practical use for risk aggregation.

Although the limitations of the standard model are clear from a theoretical perspective, it is of course of interest to determine how material these shortcomings are in practice. To answer this question, we will specify an intern model for a sample Life Insurance company.

## 3 Example Internal Model

An insurance company underwrites Life insurance and Annuities. Investments consist of equity and bonds.

The (simplified) opening balance sheet is as follows:

*Opening Balance Sheet*

| **Assets** | | | **Liabilities** | | |
|---|---|---|---|---|---|
| | | | | | |
| Equity Global | | 100 | Own Funds | | 813 |
| Equity Other | | 70 | Provisions | | 9.523 |
| Fixed rate investments | | 9.416 | | | |
| Cash | | 750 | | | |
| Total | | **10.336** | | | **10.336** |

We will include the following risks in our model: Mortality, Longevity, Lapse and Market risk. For the purpose of this analysis, individual risks are parameterised such that the shocks from the QIS 5 standard model correspond to the 99.5% confidence level of the same risks in the internal model.

By using stochastic simulation we no longer need to assume additivity of risks. Liability cash flows, interest rates, and market value of equity are generated anew in each simulated scenario, taking into account the specified dependencies between them. Subsequently, discounted cashflows, market values and own funds are determined in each simulated scenario.

For the dependencies between the risks, we consider two alternatives. In the first alternative, we adopt the correlations from the standard model, and there is no additional tail correlation[3]. In the second alternative, a high degree of tail correlation is assumed in addition to the correlations from the standard model[4].

[3] Here we use the so-called ‘Normal Copula’

[4] Using a t-copula with 1 df, an extension of the Normal copula with a high degree of tail correlation.

The results are as follows[5].

| Risk Type | Individual Risk Amount | Cumulative Risk Standard Model | Cumulative Risk Simulation Model No Tail Correlation | Cumulative Simulation Model High Tail Correlation |
|---|---|---|---|---|
| Mortality Parameter | 116 | 116 | 116 | 116 |
| Longevity | 78 | 123 | 119 | 158 |
| Mortality Catastrofe | 50 | 132 | 123 | 161 |
| Equity Global | 37 | 153 | 133 | 170 |
| Equity Other | 33 | 174 | 150 | 183 |
| Lapse | 127 | 243 | 160 | 197 |
| Interest | 193 | 382 | 296 | 344 |
| Total | **634**(sum of individual) | **382** | **296** | **344** |

The individual risk amounts are the same in all three model approaches, as the parameters of the internal model have been chosen to reflect the individual shock scenarios of the standard model.

The ‘Cumulative Risk’ is the aggregate of the risk in question and the risks in the preceding rows of the table. For example, the aggregate of Mortality Parameter, Longevity and Mortality Catastrophe risk including diversification, amounts to 132 in the standard model. The Longevity risk in isolation equals 78 in all three versions.

Total required capital in both versions of the internal stochastic simulation model are considerably lower than the standard model. If a high degree of tail correlation is assumed between all individual risks, then the total required capital in the simulation model is still considerably below that total in the standard model.

The biggest difference between the results of the standard model and both internal models is in the contribution of the Lapse risk. An increase in Lapse leads to lower Mortality and Longevity, as a smaller portfolio remains. Although the Lapse risk is large in itself, higher lapse leads to lower Mortality and Longevity risk. Therefore the contribution of the Lapse risk to the total aggregated risk remains limited.

Furthermore, the contribution of Equity risk to the aggregated risk in the internal model is considerably lower than in the standard model. The difference can be attributed to the use of a skewed, non-Normal, distribution of the risk in the internal model.

[5] Results were generated using @Risk.

**Final remarks**

In the example, both versions of the internal stochastic model give rise to a lower required capital amount than the standard model. An internal model may however also lead to a higher capital requirement, depending on risk profile and parameterisation.

For most risk types there are, almost by definition, very few or no observations of extreme outcomes available for parameterisation. Therefore the choice of shock parameters as well the degree of tail correlation will always contain an element of subjectivity. The assumption that there would be no tail correlation, however, is hard to defend given recent experience particularly with market risks

All in all, we conclude that using stochastic simulation as part of an internal model approach enhances insight into the aggregated risk profile an insurer is exposed to. This enables the actuary to improve his assessment of the capital requirement and financial position of an insurer.