Considerations on the Discount Rate in the Cost of Capital Method for the Risk Margin

Hans Waszink MSc AAG MBA
Waszink Actuarial Advisory
The Hague
The Netherlands
www.waszinkadvice.com
hans@waszinkadvice.com

May 2013

Abstract

In this article we will discuss actual and desirable properties of the Cost of Capital Method to set the Risk Margin when valuing insurance liabilities. We will show that, particularly for liabilities with very long maturities, the Cost of Capital method fails to satisfy a number of desirable properties, for example it has no upper bound related to the Capital Requirement or the maximum value of liability, and it is not invariant under the choice of time unit.

We will then show that these issues can be resolved by using the Cost-of-Capital rate as discount rate. Also we will discuss the assumptions that need to be made to justify the use of the Cost-of- Capital rate as the discount rate.

Key words: *Risk Margin, Cost of Capital, Discount Rate, Discounted Cash Flow, Market Value, Fair Value*.

Introduction

In recent years, the Cost of Capital Method (CoC) has gained popularity as a method to determine the value of so-called ‘unhedgeable' risks. Unhedgeable risks are risks that can not be fully hedged with instruments traded in an active market. This is the case for various risks borne by insurers and pension funds, such as Longevity, Mortality, P&C, etc.

The Risk Margin according to the CoC method is generally determined by the following steps:

1. Project the SCR, the *Solvency Capital Requirement* in all future periods of risk exposure.
2. Multiply the SCR by the Cost-of-Capital rate in each period.
3. Discount the amounts calculated under (2) using the risk free rate.

See for example [2].

In this article we will discuss actual and desirable properties of the CoC method. We will show that, particularly for liabilities with very long maturities, the CoC method fails to satisfy a number of desirable properties as an estimation for the market value of an insurance liability. We will then show that these issues can be resolved by using the Cost-of-Capital rate as discount rate, instead of the risk free rate. Also we will discuss the assumptions that need to be made to justify the use of the Cost- of- Capital rate as the discount rate.

This paper is outlined as follows. Section 1 gives a theoretical outline of the CoC method. In section 2, issues related to the use of the risk free rate are discussed. Section 3 sets out an approach that resolves these issues, followed by final remarks in section 4.

1 Outline of the Cost of Capital Method

In the absence of an active market in which insurance liabilities are traded, its value can be decomposed into:

- The hedgeable liability value (*HLV*), i.e. the value that can be replicated using tradeable financial instruments;
- The Risk Margin (*RM*), i.e. the value of the residual unhedgeable risk.

HLV is the market value of a portfolio of traded financial instruments that most closely approximates the liability cash flows in all feasible scenarios, so that the residual risk is minimised. The Risk Margin then reflects the value of the unhedgeable risk, that is the risk that cannot be replicated by financial instruments for which an active market exists.

The sum total of *HLV* and *RM* is the market price of the liability *MVL*:

$$MVL = HLV + RM\ .$$

The cash flows emanating from the asset portfolio with value *HLV* are such that they match the expected cash flows arising from the liability. The deviation in the cash flow created by the unhedgeable risk has expected value zero in each future period.

Hence in the scenario in which the actual liability cash flows match expected cash flows, *RM* and any investment income gained on it, will be released as profit and do not need to be used to settle the liability.

By inclusion of the Risk Margin, *MVL* is such that it approximates the Market Value, or Fair Value of the liability. Under IFRS, Fair Value is defined as follows (see [6]):

'Fair Value is the amount for which an asset could be exchanged, a liability settled, or an equity instrument granted could be exchanged between knowledgeable, willing parties in an arm's length transaction'

The Solvency II 'Level 1' legislation prescribes that the Risk Margin be determined as follows (see [2]):

'The risk margin should be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirements necessary to support the (re)insurance obligations over their lifetime.'

The latter is consistent with the IFRS definition of Fair Value. Under the Solvency II approach, the *willing party* assuming the liability is an investor providing *'eligible own funds'* in the amount of the Solvency Capital Requirement *SCR*, to enable the full run-off of the liability. A further assumption in the Solvency II approach is that the hedgeable liability value is indeed fully hedged by a replicating portfolio of assets, so that *SCR* only needs to cover unhedgeable risk

It is further assumed that the *SCR* itself is invested at the risk free rate with maturity corresponding to the lifetime of the liability. The investor requires an additional return on his investment, assumed to be a fixed percentage above the risk free rate, e.g. 6% per annum. This return is called the '*cost of capital*' and is necessary to compensate him for the risk of not (fully) receiving back the capital invested plus interest at the risk free rate. We will not comment on the suitability of the 6%, or any choice of fixed rate, although one may argue that this rate is not fixed but varies with market conditions.

The Risk Margin at any point in time is the present value of the future periodic returns on capital to be provided to the investor. If the Risk Margin were equal to zero, then the expected profit to the investor, apart from investment income over the *SCR*, would also equal zero. In this case, the cash flows released from the replicating asset portfolio plus income earned on it at the risk free rate, exactly cover the expected cash flows from the liability.

If the liability cannot be fully hedged, the cash flows from the replicating portfolio augmented by the periodic Cost- of-capital rate over the *SCR* are considered to be the risk free equivalent of the actual liability cash flow in each period. This means the holder of the liability is at any point in time assumed to be indifferent between having to pay out the actual uncertain cash flow of the liability, or the expected cash flow plus the Cost-of -Capital rate over the *SCR*.

RM is then determined as the periodic pay-out of the Cost-of-Capital-rate over the *SCR*, discounted at the risk free rate:

$$RM = \sum_{i=1}^{n} SCR(i-1) \times CoCr \,/\, [1+ r_f(i)]^i \,.$$

with: *SCR(i)* the projected *SCR* at the beginning of period *i*.

CoCr : required return on the SCR in excess of the risk-free rate.

$r_f\,(i)$: risk free rate for maturity *i* periods.

n: the number of periods until full run-off of the liability.

2 The Discount Rate within the Cost of Capital Method

We will not comment on the methods used to set the required rate of return, the risk-free rate or projections of future SCR. We recognise that there is a degree of subjectivity in how these are set, and that of all these are in fact unknown, stochastic quantities at any future point in time[1]. Instead we focus on the use of the risk free rate as the discount rate.

Given projections of future *SCR* and the assumed required return by the investor, we have found that using the risk free rate as the discount rate gives rise to the following issues:

- The Risk Margin can exceed the *SCR*, and even the theoretical maximum value of the liability.
- More generally, the Risk Margin does not reflect the present value of the expected returns from the perspective of the investor providing the required capital.
- The Risk Margin is not invariant under a change of time unit, e.g. when switching from an annual to a monthly projection the Risk Margin will change.

We discuss these findings in more detail below.

2.1 The Investor's Perspective

Suppose that at a certain point in time, an investor provides an amount of capital to an insurer, sufficient to enable the insurer to run-off a liability in compliance with all internal and external capital requirements. We will refer to this amount as *SCR*.

According to the cost of capital method, the Risk Margin *RM* represents the upfront value of the compensation that would be required to find an investor willing to provide this capital. By doing so, the investor accepts the risk of receiving an uncertain, residual return, after all obligations to insureds are met and adequate provisions are held. Alternatively, the investor could invest in a (supposedly) risk free asset with a similar maturity so that he would be certain to receive his investment back in full plus interest at the appropriate risk free rate.

It is intuitively clear that the risk to the investor is no greater than the capital he invests. He can lose no more than his investment, as he is under no obligation to provide additional funding at a later stage. Hence his upfront cost of capital can not be higher than *SCR*. If there is even a remote chance of receiving some interest or part of the invested capital back at a future date, then the of the cost of capital at present from his perspective is below the initially invested amount of *SCR*.

Moreover, if the investor received a total return on his investment of which the present value *RM* exceeded *SCR*, then that would create an arbitrage opportunity. By investing $SCR < RM$, the investor would in an efficient market be able to receive *RM*

[1] See for example [7]

upfront from another investor in exchange for all future earnings arising during the full run-off of the liability.

Some examples in which *RM* according to the CoC method, can nevertheless exceed *SCR* are as follows:

Example 1

Suppose the *SCR* remains constant over an infinitely long time horizon, and the discount rate *r* used to determine *RM* is independent of the maturity. Also, assume that no investment income is earned on the *SCR*. This is obviously a simplified example, and should be considered as a limiting case:

$$RM \quad =$$

$$CoCr \sum_{i=1}^{\infty} SCR\ (i\text{-}1)/\ (1+r)^{i} \quad =$$

$$CoCr\ SCR \sum_{i=1}^{\infty} 1\ /\ (1+r)^{i} \quad =$$

$$CoCr\ SCR\ /\ r.$$

The last equality follows from the well known equality:

$$\sum_{i=1}^{\infty} 1\ /\ x^{i} = 1/(x-1) \text{ for } x > 1.$$

Hence if $CoCr > r$ then $RM > SCR$. For example if $CoCr$ = 6% and r= 2% then the Risk Margin equals three times the SCR.

We conclude that in this case, for the condition $RM \leq SCR$ to hold, we must have $r \geq CoCr$.

Furthermore, in this case it is directly clear that regardless of the choice of *CoCr*, the cost of capital to the investor from en economic perspective is simply *SCR*. The investor provides an amount of *SCR* at present, none of which will ever be returned as the capital remains in the company indefinitely and no investment income is earned.

For *RM* to be equal to *SCR* in this case, we must have: $r = CoCr$.

Example 2

The assumption that the *SCR* remains constant indefinitely can be relaxed. In the next example we assume that the *SCR* is not constant but declines exponentially at rate *d* per annum, hence:

$$SCR(t) = (1-d) \times SCR(t-1) \text{ for all } t=1,2,3,\ldots.$$

Now we have:

$$RM =$$

$$CoCr \sum_{i=1}^{\infty} [SCR\,(0) \times (1\text{-}d)^{i-1} / (1+ r\,)^{i}] \qquad =$$

$$CoCr\;\; SCR\,(0) \times \frac{1}{1-d} \times \sum_{i=1}^{\infty} [(1\text{-}d) / (1+ r\,)]^{i} \qquad =$$

$$CoCr \times SCR(0) \times \frac{1}{1-d} \times [(1\text{-}d)/(r+d)] \qquad =$$

$$CoCr \times SCR(0)/(r+d).$$

So in this example we have: $RM > SCR(0)$ if $CoCr > r + d$.

For example, if r=2%, d =3% and $CoCr$=6% then RM = 120% × SCR(0) and $SCR(0) > SCR(t)$ for any t >0.

In example 1, the *RM* is three times greater than the liability in the worst case scenario used to determine *SCR*. There is no upper bound to the Risk Margin linked to the maximum coverage provided in the portfolio of risks for which *RM* is held. Therefore *RM* may exceed the total insured value in the portfolio, or the chance that the actual liability will be larger than *HLV* + *RM* margin may be extremely remote.

A value of *HLV* + *RM* in excess of the maximum possible loss, or a loss occurring with an extremely low probability is not an adequate reflection of the ex-ante value of the risk, except possibly where the range of outcomes of the risk is extremely small.

2.2 Invariance under choice of time unit

The chosen time unit in the Cost of Capital Method is often a single year. This is however an arbitrary choice, as the actual risk does not depend on the choice of time unit for modelling or reporting purposes. Also, one should be able to recalculate the Risk Margin in a consistent manner at any point in time and not just at an entire multiple of the chosen unit time interval.

One may argue that a narrower time unit will yield a more accurate assessment of the Risk Margin, as the development of *SCR* over time can be followed more closely. We therefore first consider the case in which *SCR* stays constant over the entire run-off period, at the end of which it immediately drops to 0. As an example, assume the run-off period is ten years:

$SCR(t) = 100$ for $0 \le t < 10$ and
$SCR(t) = 0$ for $10 \le t$,

with t the number of years after determining the Risk Margin.

Further assume $CoCr$=6%, r=2% both per annum.

When choosing a time unit of one year, RM equals:

$$RM = 6\% \sum_{i=1}^{10} 100/(1.02)^i = 53.90 .$$

If we use a ten year time unit, we first need to convert the 6% required return and the 2% risk free rate to the equivalent rates over a ten year period:

Ten year required rate = $1.06^{10} - 1 = 79.08\%$
Ten year risk free rate = $1.02^{10} - 1 = 21.90\%$.

Now the Risk Margin RM equals:

$$RM = 100 \times 79.08\%/(1+21.90\%) = 64.88.$$

When switching to a ten year time unit, the Risk Margin increases from 54 to 65.

More generally, as shown in appendix I, the only choice of discount rate for which RM is invariant under the choice of time unit is: $r = CoCr$.

The difference between the two results can be explained as follows. By discounting the periodic future payments to the investor at the risk free rate, it is implicitly assumed that after receipt of RM, the Risk Margin is invested in an asset yielding the risk free rate. The annual payments of $CoCr \times SCR$ are paid out of the accumulated total of RM and the investment income earned on it.

By switching to a ten year time unit, the investor chooses to postpone all ten annual payments in the amount of $CoCr \times SCR$, to the equivalent amount at the end of the ten year period. The required return is 6% per annum, while the accumulated funds available for to provide this return are only earning the risk free rate of 2%. The shortfall must be compensated by having a higher amount of RM to start with than in the case annual payments are made.

Assumption of Constant SCR

The assumption that SCR remains constant over a fixed period and is then released at once may seem unrealistic. However, as shown in appendix II, any pattern of future *SCRs* in discrete time can be written as a linear combination of *SCRs* with such as pattern.

The graph below shows an example of a stepwise decreasing SCR pattern, represented as the sum of multiple SCR projections $SCRP$ that are all constant up to a fixed, but different, point in time.

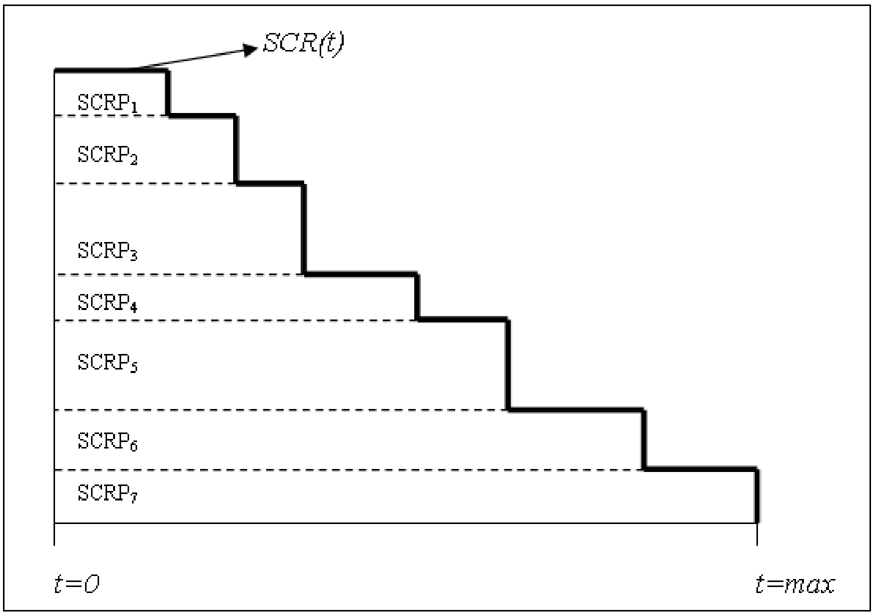

Figure 1: decomposition of SCR into constant segments

The Risk Margin for any *SCR* pattern can also be written as a linear combination of Risk Margins for *SCR* projections that are each constant up to a fixed point in time (see also appendix II). It follows that the requirement *r=CoCr* to achieve invariance under the choice of time unit, extends to any projection pattern of future *SCRs*.

2.3 Comparison with Discounted Cash Flow Methodology

A traditional method in Corporate Finance Theory to determine the present value of a set of risky cash flows is *'Discounted Cash Flow analysis'* (DCF). In DCF, future expected cash flows are discounted by a rate reflecting the risk in those cash flows. A riskier investment requires a higher discount rate.

In the case where we are valuing a liability, the direct application of DCF would give counterintuitive results. A riskier liability would generally generate a lower present value as future cash outflows are discounted at a higher rate. This is a well known drawback of DCF methods. When valuing an investment project with negative NPV, increasing the discount rate can make its NPV less negative so that the value of the investment increases.

However, DCF can still be applied in the following way when viewing the run-off of a liability as an investment opportunity:

- An investor accepts an insurance liability from another party and receives an amount equal to *HLV* to cover the expected cash flows arising from the liability. He invests the entire amount in the replicating portfolio with value *HLV*.
- The investor immediately provides an amount of capital equal to *SCR*. For now we assume that the *SCR* remains constant over the lifetime of the liability, possibly until infinity. For the investor, *SCR* represents a cash outflow as it

will be held in the insurance company and he will not have access to it until the liability is fully run off.

- Once the liability has been fully run-off, any remaining capital is returned to the investor. As the DCF method projects best estimate cash flows, the full amount of *SCR* will be returned to the investor at the end in the scenario used for valuation.
- Assume the risk free rate for all maturities equals 0. The rate of return required by the investor now equals *CoCr* and no investment income is generated over *SCR*. An alternative scenario where the risk free rate is positive, will be considered in the next paragraph.

First we consider the case in which the liability is fully run-off of over a single period. The cash flow projection from the investor's point of view is as follows:

t=0: - *SCR*
t=1: *SCR*

An amount *SCR* is invested at t=0 and returned at t=1.

Assuming a risk free rate equal to 0, the net present value (NPV) of this investment under the DCF method is:

$$NPV =$$

$$-SCR + SCR/(1+CoCr) =$$

$$[-SCR \times (1+CoCr) + SCR]/(1+CoCr) =$$

$$-SCR \times CoCr/(1+CoCr).$$

The opposite of the *NPV* represents the cost of capital. It is the present value of the cash flows from the investor's perspective based on his required rate of return. Although the *NPV* is negative, it is decreasing, i.e. becoming more negative, for higher values of *CoCr*. The reason is that all future cash flows are positive, the only negative cash flow is at t=0 so its present value is unaffected by the required rate of return.

We conclude that in this case, the upfront cost of capital is, $CoCr\times SCR$ discounted at, again, *CoCr*. For the multi period equivalent with constant *SCR*, we can proceed in two ways:

- Convert the one period rate *CoCr* into a multi-period rate: $(1+CoCr)^n-1$, for any positive value of n.
- For a positive integer value of n, project the periodic amounts $CoCr\times SCR$ and discount them at the one period rate CoCr.

It can be verified mathematically that for positive integer values of n, both these methods generate identical results, see appendix III.

Finally, to allow for other patterns of the *SCR* than constant values up to a fixed point, we can again write any pattern of *SCR* development as a linear combination of constant *SCRs* up to separate fixed points, as shown in appendix II.

We conclude that for any projection pattern of the *SCR*, the application of DCF to determine the cost of capital also requires using *CoCr* as the discount rate if the risk free rate equals zero.

2.4 Investment income on SCR

In the previous paragraph, we have assumed a risk free rate of zero. If the risk free r_f is constant and greater than zero, and the *SCR* earns the risk free rate whilst being invested, then the required rate of return becomes $r_f + CoCr$. Cash flow projections are now as follows:

t=0: $- SCR$
t=1: $SCR \times (1+r_f)$

so that under the DCF method:

$NPV =$

$-SCR + SCR \times (1+r_f)/(1+r_f + CoCr) \quad =$

$-SCR \times (1+r_f+CoCr-1-r_f)/(1+r_f + CoCr) \quad =$

$-SCR \times CoCr\ /(1+r_f + CoCr).$

In this case, the required return $SCR \times CoCr$ is discounted at the rate of $r_f + CoCr$, higher than in the previous paragraph so that the resulting Risk Margin is lower. Assuming $r_f = 0$ therefore provides an upper bound for the Risk Margin with regard to the actual risk free rate.

Only if the future expected returns $CoCr \times SCR$ were risk free should the risk free interest rate be used as discount rate to determine their NPV. This is however clearly not the case, as the *SCR* serves as buffer and may never be returned to the investor.

If, on the other hand, no investment income is earned on the *SCR* but a return equal to $r_f + CoCr$ is required, then *CoCr* should be replaced by $CoCr + r_f$ in both numerator and denominator in the NPV formula in paragraph 2.3, so the Risk Margin would be somewhat higher.

Furthermore, we note that if $r_f > 0$, also the DCF method is not invariant under a change of time unit as in that case the discount rate $r_f + CoCr > CoCr$. This is a violation the condition formulated in 2.2 that the discount rate must equal *CoCr* to obtain invariance under the choice of time unit.

3 Derivation of SCR and RM from CoC-Method Assumptions

In the previous sections, we have shown that the discount rate in the CoC-method needs to be equal to *CoCr*, in order to satisfy a number of practically desirable properties. In this section, we derive the formula for the Risk Margin directly from assumptions underlying the CoC-method to arrive at the same result.

One might be inclined to think that, by discounting the future payments $SCR \times CoCr$ at the rate *CoCr*, it would be necessary to invest the Risk Margin in a risky asset. We will show that this is not the case.

As before, let *SCR* be the amount of capital required to enable the insurer to run-off a liability in compliance with all internal and external capital requirements.

The key underlying assumption we will use to derive *RM* is the following:

> *SCR at any point in time is the present value of the worst possible deviation from the best estimate value of the liability that can occur during its run-off.*

We derive this assumption from the fact that *SCR* is the capital required to support the run-off of the risk, prior to setting the Risk Margin. Even if the present value of the liability could deviate from expected by more than *SCR*, the possibility of such an event is assumed not to have an impact on the Risk Margin. Hence, for the ex-ante valuation of the risk, the likelihood of an unexpected loss in excess of *SCR* is deemed negligible.

This assumption is also consistent with the Solvency II prescribed approach for the CoC method outlined in chapter 1, which states that *SCR* is the amount of own funds '*necessary to support the (re)insurance obligations over their lifetime*'.

Note that this assumption holds regardless of the method that was used to determine *SCR*. It may have been calibrated at a 99,5% confidence level over a single year, or a lower confidence level over the full run-off period of the liability. Also it may have been based on a VaR or a TVar measure. But the CoC-method does not distinguish between the manner in which the *SCR* was determined in the first place, but only ensures that a sufficient *expected* return over *SCR* can be made.

Hence the starting point of the CoC-method is that exactly the amount *SCR* is available to cover unexpected losses. This would be the case if the investor received a return of $CoCr \times SCR$ at the end of each period. However, as *RM* is provided upfront and forms part of the balance sheet liability, it also forms part of the buffer available to cover unexpected losses. Therefore the total buffer available to cover unexpected losses is $SCR+RM$.

If a loss in the amount of *SCR* were to occur, then no further unexpected losses could occur according to the assumption specified above. Hence, in this case the entire buffer *RM* would become available to cover the loss in the amount of *SCR*. After that, no Capital or Risk Margin would need to be held as the worst case loss had already occurred.

One might argue that this assumption is not realistic as *RM* may show volatility due to changes in market risk appetite or otherwise. Also additional capital may be required after the 'worst case' shock is suffered to support the further run-off of the liability. Therefore additional prudence may be sought when setting the Risk Margin.

However, as shown in examples 1 and 2 of section 2, for liabilities with very long maturities the additional prudence that arises as a result of using the risk free for discounting may be excessive. If additional prudence is desirable, this could also be achieved by increasing *SCR* or *CoCr*. In this way, a consistent set of assumptions is used, and *RM* will still have *SCR* as upper bound.

We will therefore continue to assume that *SCR*, however determined, is the highest possible change in the present value of the liability. Hence only an amount *SCR' = SCR – RM* put up by the investor is actually at risk. Even if he still (has to) put up the entire amount of *SCR* as capital, an amount *RM* thereof is actually not exposed to any risk. Therefore the investor has no reason to require anything more than a risk free return on this part of his investment.

We will now first consider the one period case to derive the Risk Margin. In this case, the liability has fully run-off after one period and all remaining funds are returned to the investor after settlement of the liability.

Assuming a risk free interest rate $r_f = 0$, *SCR'* and *RM* satisfy the following equations:

$$SCR' + RM = SCR$$
$$RM = CoCr \times SCR' \ .$$

The first equation indicates the total of *SCR'* and *RM* is sufficient to the cover the worst case loss of *SCR*. The second equation indicates that *RM* needs to cover the required return on *SCR'* at the time the liability has run off. As $r_f = 0$, the total required rate of return is *CoCr*, and no investment income is earned during run-off.

This set of equations has the following solution:

$$SCR' = SCR/(1+CoCr)$$

$$RM = SCR \times CoCr/(1+CoCr) \ .$$

RM equals the *CoCr* rate over *SCR*, discounted at again, *CoCr*.

In the case that $r_f > 0$, the required return is $CoCr + r_f$ and we get:

$$SCR' + RM = SCR$$

$$RM + SCR \times r_f = (CoCr + r_f) \times SCR' \ .$$

The first equation is unchanged- the total of *SCR'* and *RM* still needs to cover the worst case loss of *SCR*. The left part of the second equation is the total return available for the investor in the expected scenario, equal to *RM* plus investment income over *SCR* at the risk free rate. The right part of the second equation indicates the return required by the investor, equal to $CoCr + r_f$ over his investment *SCR'*.

These two equations have the following solution:

$$SCR' = SCR \times (1+r_f)/(1+r_f + CoCr)$$

$$RM = SCR \times CoCr/(1+r_f+CoCr).$$

For both r_f=0 and r_f >0, these results are identical to the results of the DCF method shown in 2.3 and 2.4. We note that also here, *RM* is only invariant under the choice of time unit if r_f=0 according to the result of paragraph 2.2, and that assuming r_f=0 provides an upper bound for the Risk Margin with regard to the choice of the risk free rate r_f.

The multi-period case if $r_f = 0$ can again be derived by replacing *CoCr* with its multi-period equivalent. In case *SCR* remains constant over a period of length *t*, with *t* any positive value, replacing *CoCr* by $(1+CoCr)^t-1$ in the formule above for *RM* gives:

$$RM =$$

$$SCR \times [(1+CoCr)^t-1]/(1+CoCr)^t \quad =$$

$$SCR \times [1- 1/(1+CoCr)^t] \quad =$$

$$SCR \times CoCr \times \sum_{i=1}^{t} 1/(1+CoCr)^i ,$$

with the last equality for integer values of *t*, as shown in appendix III.

We can now also derive the development pattern of *SCR'* and *RM* as the liability runs off. Let *RM(t')* be the Risk Margin at time $t' \leq t$ and $SCR'(t') = SCR - RM(t')$. The amount of time outstanding until *t* at time *t'* is *t-t'* so that

$$SCR'(t') = SCR/(1+CoCr)^{t-t'} \text{ and}$$

$$RM(t') = SCR \times (1- 1/(1+CoCr)^{t-t'}).$$

Hence *SCR(t')* grows exponentially with *t'* at a rate of *CoCr* per time unit, and $SCR(t') + RM(t') = SCR$ for all $t' \leq t$. Note that this formula holds for all positive *t'* and not just for integer values.

At the end of the run-off period, when *t'*=*t*, the investor will receive an amount $SCR'(t') = SCR$ in the scenario that actual liability cash flows equal best estimate cash flows over the entire period. In this scenario he will therefore have earned exactly the required return of *CoCr* per period.

Alternatively, the investor may, under the same scenario, transfer the liability to another investor at any other time *t'* in exchange for the Risk Margin *RM(t')*. The 'other' investor will then provide an amount of capital equal to *SCR'(t')* to the original investor. In this way, the latter will earn a return of *CoCr* per period over the period from 0 to *t'*.

As before (see appendix II), we can extend this method by taking linear combinations of *SCR* projections that are all constant up to a fixed point, which allows us to use any pattern of *SCR* projections.

This allows us to write *RM* for any pattern of SCR projections over *n* periods as:

$$RM = CoCr \times \sum_{i=1}^{n} 1/(1+CoCr)^{i} \times SCR(i-1)\ .$$

4 Final Remarks

The Cost of Capital method is an intuitive and logical method to estimate the market value of a risk. However, the use of the risk free rate for discounting in the CoC formula gives rise to a number of undesirable properties of the Risk Margin. In particular, the Risk Margin is not invariant under the choice of time unit, and has no upper bound related to the required capital or the maximum value of the risk.

A precise formulation of the assumptions underlying the CoC method is required to set the constants and parameters used. By assuming the maximum unexpected loss to be equal to the *SCR*, we have shown that the Cost of Capital rate is an appropriate rate for discounting, and resolves the issues discussed. Also, it was shown that the implicit assumption of a zero risk free rate when using the Cost of Capital rate as the discount rate, provides an upper bound for the Risk Margin with regard to the actual risk free rate.

It is evident that some of the underlying assumptions of the method are only true by approximation, in particular the assumption that unexpected losses are limited by the amount of *SCR*. Additional prudence can be introduced by increasing *SCR* or the Cost of Capital rate or by lowering the discount rate. However, for liabilities with very long maturities, the use of the risk free rate for discounting can give rise to inconsistent and counterintuitive results, such as a Risk Margin in excess of the required capital.

References

1. Brealey, R., Myers, S., Marcus, A.,1995. The Fundamentals of Corporate Finance. McGraw-Hill.

2. CEIOPS, 2009. Final CEIOPS' Advice for Level 2 Implementing Measures on Solvency II: Technical Provisions – Article 86 (d) Calculation of the Risk Margin.

https://eiopa.europa.eu/fileadmin/tx_dam/files/consultations/consultationpapers/CP42/CEIOPS-L2-Final-Advice-on-TP-Risk-Margin.pdf

3. Chief Risk Officer Forum, 2006. A market cost of capital approach to market value margins.

http://www.thecroforum.org/a-market-cost-of-capital-approach-to-market-value-margins/

4. Filipovic, D., Vogelpoth, M., 2007. A note on the Swiss Solvency Test risk measure.

http://infoscience.epfl.ch/record/148556/files/A%20note%20on%20the%20Swiss%20Solvency%20Test%20risk%20measure.pdf

5. IAA, ad hoc Risk Margin Working Group, 2009. Measurement of Liabilities for Insurance Contracts.

http://www.actuaries.org/LIBRARY/Papers/IAA_Measurement_of_Liabilities_2009-public.pdf

6. IFRS, 2009. Fair Value Measurement.

http://www.ifrs.org/Conferences-and-Workshops/Documents/2009IFRSconferenceSeoulFVM.pdf

7. Pelsser, A., 2011. Pricing in Incomplete Markets. Netspar Panel Papers.

http://arno.uvt.nl/show.cgi?fid=114741

8. Waszink, H., 2009. Comments from Waszink Actuarial Advisory on CEIOPS CP 42 Risk Margins.

https://eiopa.europa.eu/fileadmin/tx_dam/files/consultations/consultationpapers/CP42/Comments-received-from-WAA-on-Consultation-Paper-42-09.pdf

Appendix I

Let $SCR(t)$=1 for t =0,1,2,…,T-1

$SCR(t)$ =0 for t=T,T+1, T+2,… for some positive integer T.

Assuming a constant risk free rate r >0, the risk margin RM is:

$$RM = CoCr \sum_{t=1}^{T} \frac{1}{(1+r)^t}$$

$$= \frac{CoCr}{r_f}\left[\frac{(1+r)^T - 1}{(1+r)^T}\right].$$

This follows from the equality $r\sum_{t=1}^{T}\left(\frac{1}{1+r}\right)^t = 1 - \frac{1}{(1+r)^T}$, as shown in Appendix III with $CoCr$ replaced by r.

When switching to a time unit of length T, the required return $CoCr_T$ and the risk free rate r_T are:

$$CoCr_T = (1+CoCr)^T - 1$$
$$r_T = (1+r)^T - 1$$

and the resulting risk margin RM_T is:

$$RM_T = CoCr_T/(1+ r_T)$$

$$= \frac{(1+CoCr)^T - 1}{(1+r)^T}.$$

First set T=2. For $RM_T = RM$ to hold, we need to have that:

$$\frac{(1+CoCr)^2 - 1}{(1+r)^2} = \frac{CoCr}{r}\left[\frac{(1+r)^2 - 1}{(1+r)^2}\right] \Leftrightarrow$$

$$(1+CoCr)^2 - 1 = \frac{CoCr}{r}[(1+r)^2 - 1] \Leftrightarrow$$

$$r(2\,CoCr + CoCr^2) = CoCr(2r + r^2) \Leftrightarrow$$

$$r\,CoCr^2 = CoCr\,r^2.$$

As we have required $r >0$, this equality only holds in the trivial case $CoCr=0$ or $r = CoCr$.

Furthermore, it is easily verified that $RM_T = RM$ for all positive integer values of T if $r = CoCr$.

In the case that $r = 0$, which we have excluded so far, we have

$$RM = T \times CoCr$$

$$RM_T = (1+CoCr)^T - 1.$$

In this case, $RM = RM_T$ if and only if $CoCr = 0$.

This proves that $RM_T = RM$ if and only if $r = CoCr$.

Appendix II

Define

$$SCRD(t) = SCR(t) - SCR(t+1) \text{ for all } t=0,1,2,...$$

$$SCR(\infty) = \lim_{t \to \infty} SCR(t).$$

Then:

$$SCR(t) = \Sigma_{t' \geq t} SCRD(t') + SCR(\infty).$$

Define $SCRP_k(t)$ with $k=0,1,2,\ldots$, as:

$$SCRP_\infty(t) = SCR(\infty)$$

$$SCRP_k(t) = SCRD(k) \text{ if } 0 \leq t \leq k$$

$$SCRP_k(t) = 0 \text{ if } t > k.$$

Then we can write $SCR(t)$ as:

$$SCR(t) = \sum_{k=0}^{\infty} SCRP_k(t)$$

$SCR(t)$ is now the sum of the $SCRP_k(t)$ which are all constant up to a fixed point k and then drop to 0.

Let $\{SCR_i\}$ be a set of projected SCR amounts: $SCR_i(t)$, $t = 0,1,2,\ldots$
Let RM_i be the risk margin following from $\{SCR_i\}$: $RM_i = RM_i(\{SCR_i\})$

As RM_i is a linear function of $\{SCR_i\}$, it follows that:

$RM(\{SCR_{i\&j}\}) = RM_i(\{SCR_i\}) + RM_j(\{SCR_j\})$

with $SCR_{i\&j}(t) = SCR_i(t) + SCR_j(t), t=0,1,2,\ldots$

For any set of SCR projections $\{SCR\}$, we can write RM as:

$RM =$

$RM(\{SCR\}) =$

$$RM\left(\sum_{k=0}^{\infty} \{SCRP_k\}\right) =$$

$$\sum_{k=0}^{\infty} RM(\{SCRP_k\}).$$

We conclude that also the risk margin RM can be written as the sum of risk margins of SCR projections that are all constant up to a fixed point.

NB if $SCR(t)$ is increasing over any time interval, then $SCRPD(t)$ will become negative for some value of t but all equations still hold.

Appendix III

Instead of $NPV = -SCR \times CoCr/(1+CoCr)$, we get, with SCR=1 and $CoCr$ replaced by $(1+CoCr)^n - 1$:

$NPV =$

$-[(1+CoCr)^n - 1]/(1+CoCr)^n =$

$-[1 - 1/(1+CoCr)^n] =$

$$-CoCr\,[1/CoCr - \frac{1}{(1+CoCr)^n}\,1/CoCr\,] =$$

$$-CoCr\,[1/CoCr - \frac{1}{(1+CoCr)^n}\sum_{i=1}^{\infty}\frac{1}{(1+CoCr)^i}\,] =^{*}$$

$$-CoCr\,[\sum_{i=1}^{\infty}\frac{1}{(1+CoCr)^i} - \sum_{i=n+1}^{\infty}\frac{1}{(1+CoCr)^i}\,] =$$

$$-CoCr\sum_{i=1}^{n}\frac{1}{(1+CoCr)^i}\ .$$

* Using the equality $\sum_{i=1}^{\infty}\frac{1}{(1+CoCr)^i} = 1/CoCr$.